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Flow Map Parameterization Methods for Invariant Tori in Quasi-Periodic Hamiltonian Systems*

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Abstract. The aim of this paper is to present a method to compute parameterizations of partially hyperbolic invariant tori and their invariant bundles in nonautonomous quasi-periodic Hamiltonian systems. We generalize flow map parameterization methods to the quasi-periodic setting. To this end, we introduce the notion of fiberwise isotropic tori and sketch definitions and results on fiberwise symplectic deformations and their moment maps. These constructs are vital to work in a suitable setting and lead to the proofs of "magic cancellations" that guarantee the existence of solutions of cohomological equations. We apply our algorithms in the elliptic restricted three body problem and compute nonresonant 3-dimensional invariant tori and their invariant bundles around the L_1 point.

Key words. invariant tori, quasi-periodic Hamiltonian systems, parameterization method, ERTBP, KAM

MSC codes. 37J40, 37C60, 37M21, 34C45

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1. Introduction.

The study of invariant manifolds constitutes the center piece in understanding dynamical systems. It is a rather natural first approach—and often the only hope—to unveil the qualitative behavior of a time-evolving system. Besides the intrinsic interest of invariant manifolds, such structures have found their "real-world" analogues in celestial mechanics, astrodynamics and mission design, plasma physics, semiclassical quantum theory, magnetohydrodynamics, neuroscience, and the list goes on. In particular, celestial mechanics has a long-held tradition in considering such objects, especially periodic orbits and invariant tori carrying quasi-periodic motion, and is in fact one of the main fields that promoted their rigorous and numerical study. Astronomers have used perturbative techniques for centuries in the form of formal series of dubious convergence due to the existence of the so-called small

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