Periodic orbits for an autonomous version of the Duffing-Holmes oscillator

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Abstract

In the autonomous Duffing-Holmes oscillator was detected numerically the existence of periodic orbits. Here we prove analytically that such periodic orbits exist using the Hopf bifurcation theory. We also provide the exact bifurcation value where the Hopf bifurcation takes place.

1 INTRODUCTION AND STATEMENT OF THE RESULTS

A classical nonlinear differential system with chaotic dynamics is the Duffing-Holmes nonautonomous oscillator

$$\ddot{x} + b\dot{x} - x + x^3 = a \sin \omega t. \tag{1}$$

This oscillator has been studied intensively, see [1,2,3,4,5,6,7,8,9,10,11,12,13,14]. In article [10], the authors considered the following autonomous version of Duffing-Holmes oscillator

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= x + ay - bz - x^3, \qquad a, b, c \in \mathbb{R}, \\ \dot{z} &= c(y - z). \end{aligned} \tag{2}$$

They constructed a specific electrical circuit to imitate solutions of the oscillator (2) and obtained the simulation and experimental results and the corresponding electronic device. The irregular behavior of time series of the differential system (2) can lead to chaos.

In the reference [8], the authors provided the analysis of the differential system (2) around the specific values of parameters $a = 19/10$, $b = 5/2$ and $c \in (2.5, 3.85) \subset \mathbb{R}$. In this case system (2) has exactly three equilibrium points at $(-1, 0, 0)$, $(0, 0, 0)$ and $(1, 0, 0)$. They examined the characteristics of these points under the change of the variable c from 2.55 to 3.85 and got