PERSISTENCE AND ZERO–HOPF EQUILIBRIUM IN THE TRITROPHIC FOOD CHAIN MODEL WITH HOLLING FUNCTIONAL RESPONSE

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Abstract. In this paper we analyze the persistence of three species in a three-level food chain model. We characterize when such a model exhibits a zero-Hopf equilibrium point. We show that the existence of a zero-Hopf equilibrium point only is possible if the functional responses in the model are of type Holling III or IV.

1. Introduction and statements of the main results

The interguild predation takes place when two species of predators compete for the same population of prey, and at the same time, one of them feeds on the other predator species, or, each one is prey for the other. This topic has received a lot of attention due to its impact on the community structure, as well as its use in biological control. The main objective is to investigate, through mathematical modeling, the behavior and environment mechanisms that lead a intraguild predation system to a dynamic of coexistence, either through an equilibrium point or stable limit cycles under robust conditions.

A special case of intraguild predation is the tritrophic food chain in which a prey, a predator and a super-predator participate. The study of this type of food chains has been a subject of research for a long time. For more details about the intraguild predation you can see the article of Hold and Polish [15]. The paper of Freedman and Waltman [14] of 1977 is an excellent reference about the analysis of the interaction between three-species food-chain model. These authros examine the questions of survivability of all species, utilizing a relatively general model. They establish criteria of boundlessness and stability, three special cases of the model are analyzed. One special case is the Lotka-Volterra (where they gave necessary and sufficient conditions for persistence), the other two cases are, the Lotka-Volterra predation with a carrying capacity at the lowest level, and a mixed Lotka-Volterra with Holling functional responses (at different levels) with a carrying capacity at the lowest level. The results are related with the existence of an equilibrium point in the positive octant and its stability.

The following system is the model simulating a food chain, where $x(t)$ is the size of lowest trophic species or prey, $y(x)$ is the number of middle trophic level species or first predator, and $z(t)$ is the number of highesttrophic level species or second predator,

$$\begin{aligned} \dot{x} &= h(x) - f(x)y, \\ \dot{y} &= c_1 f(x)y - g(y)z - d_1 y, \\ \dot{z} &= z(c_2 g(y) - d_2). \end{aligned} \tag{1}$$

The function $h(x)$ represents the reproduction rate of the resource (the prey), $f(x)$ is a functional response of the predator y over the prey, and $g(y)$ is the functional response of the superpredator z over the predator y. All parameters c_1, c_2, d_1 and d_2 are positive real

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