SIAM J. APPLIED DYNAMICAL SYSTEMS
Vol. 23, No. 1, pp. 98–126

Semianalytical Computation of Heteroclinic Connections Between Center Manifolds with the Parameterization Method*

Miquel Barcelona†, Alex Haro‡, and Josep-Maria Mondelo†

Abstract. This paper presents a methodology for the computation of whole sets of heteroclinic connections between isoenergetic slices of center manifolds of center $\times$ center $\times$ saddle fixed points of autonomous Hamiltonian systems. It involves (a) computing Taylor expansions of the center-unstable and center-stable manifolds of the departing and arriving fixed points through the parameterization method, using a new style that uncouples the center part from the hyperbolic one, thus making the fibered structure of the manifolds explicit; (b) uniformly meshing isoenergetic slices of the center manifolds, using a novel strategy that avoids numerical integration of the reduced differential equations and makes an explicit three-dimensional representation of these slices as deformed solid ellipsoids; (c) matching the center-stable and center-unstable manifolds of the departing and arriving points in a Poincaré section. The methodology is applied to obtain the whole set of isoenergetic heteroclinic connections from the center manifold of L_2 to the center manifold of L_1 in the Earth-Moon circular, spatial restricted three-body problem, for nine increasing energy levels that reach the appearance of halo orbits in both L_1 and L_2. Some comments are made on possible applications to space mission design.

Key words. parameterization method, heteroclinic connections, invariant tori, libration point orbits, RTBP, center manifolds

MSC codes. 37C29, 37N05, 37M05, 70F07

DOI. 10.1137/23M1547883

1. Introduction.

Heteroclinic connections play an important role in the description of dynamical systems from a global point of view. In this work we address the systematic computation of heteroclinic connections between center manifolds of fixed points of center $\times$ center $\times$ saddle type of autonomous, 3-degrees-of-freedom Hamiltonian systems.

Our interest in this case comes from applications in astrodynamics, namely to libration point space missions. In these missions, spacecraft are sent to orbits that stay close to the fixed (in a rotating frame) points L_1, L_2 of the spatial, circular restricted three-body problem (RTBP). This model describes the dynamics of a massless particle under the attraction of

*Received by the editors January 19, 2023; accepted for publication (in revised form) by J. Mireles-James August 18, 2023; published electronically January 4, 2024.
https://doi.org/10.1137/23M1547883
Funding: This work has been supported by the Spanish grants MCINN-AEI PID2020-118281GB-C31 and PID2021-125535NB-I00, the Catalan grant 2021 SGR 0013, the Spanish State Research Agency, through the Severo Ochoa and María de Maeztu Program for Centers and Units of Excellence in R&D (CEX2020-001084-M).
†Departament de Matemàtiques and CERES-IEEC, Universitat Autònoma de Barcelona. Av. de l'Eix Central, Edifici C, Bellaterra 08193, Spain (miquel.barcelona@uab.cat, josemaria.mondelo@uab.cat).
‡Departament de Matemàtiques i Informàtica, Universitat de Barcelona, Gran Via 585, Barcelona 08007, Spain, and Centre de Recerca Matemàtica, Bellaterra 08193, Spain (alex.haro@ub.edu)