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Stability of Equilibrium Points in the Spatially Restricted $N+1$-Body Problem with Manev Potential*

Mauricio Ascencio†, Esther Barrabés‡, Josep M. Cors§, and Claudio Vidal¶

Abstract. We study the dynamics of an infinitesimal mass under the gravitational attraction of $N-1$ primaries arranged in a planar ring configuration plus the influence of the central mass with a Manev potential $(-1/r+e/r^2)$, $e\neq 0$, where e is a parameter related to the oblaticity or radiation source (according to the sign of the parameter e). Specifically, we investigate the relative equilibria of the infinitesimal mass and their linear stability as functions of the parameter e and the mass parameter β, the ratio of mass of the central body to the mass of one of the $N-1$ remaining bodies. We also prove the nonexistence of binary collisions between the central body and the infinitesimal mass.

Key words. restricted $N+1$-body problem, Manev potential, equilibrium points, stability

MSC codes. 70F10, 70F15, 37C25

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1. Introduction.

The two-body problem with a quasi-homogeneous potential of the form $-(a/r-e/r^2)$, where r is the distance between the two bodies, and a, e are real constants, was considered by Newton in his work Philosophiae Naturalis Principia Mathematica (Book I, Article IX, Proposition XLIV, Theorem XIV, Corollary 2). One of the reasons to add the term e/r^2 to the gravitational attraction $(-a/r)$ was the impossibility of explaining the Moon's apsidal motion within the framework of the inverse-square force law, although the model was abandoned in favor of the classical Newtonian potential. Manev in 1924, [15], proposed a similar corrective term in order to maintain classical mechanics and offering at the same time good explanations of the observed phenomena as in the relativity theory. For instance, when a is positive and e is negative, the corrective term is good enough to explain the perihelion advance of Mercury.

In this work we consider the motion in a three-dimensional space of an infinitesimal mass P under the gravitational attraction of $N=n+1$ point masses, P_0, P_i, $i=1,\dots,n$, called *primaries*. We assume that the potential generated by the primary P_0 is a Manev potential $(-1/r+e/r^2)$ with parameter e, and that the gravitational attraction due to P_i, $i=1,\dots,n$,

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†Universidad Del Bio-Bio, Concepcion, Chile (maascencio@ubiobio.cl).
‡Universitat de Girona, Girona, Spain (esther.barrabes@udg.edu).
§Universitat Politécnica de Catalunya, Manresa, Spain (josep.m.cors@upc.edu).
¶Matemáticas, Universidad del Bio-Bio Facultad de Ciencias, Concepcion, Concepción, VIII-Región, Chile (clvidal@ubiobio.cl).