GLOBAL PHASE PORTRAIT OF A ROLLING MOTION OF SHIPS EQUATION

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ABSTRACT. This paper deals with the study of the global dynamics of a model for the rolling motion of ships in random beam seas, which is described by the second order differential equation

$$\ddot{x} + (2\mu + \delta x^2)\dot{x} + \omega^2 x - \frac{1}{a^2}x^3 = 0,$$

where $0 < \mu \ll 1$, $0 < \delta < 1$, $\omega = 1$ and $0 < a < 3$. We give the complete description of its dynamics in the Poincaré compactification of $\mathbb{R}^2$.

1. Introduction and statement of the result

The study of the problem of the motion of ships in the sea consists of solving the equations of the dynamic balance of forces and moments. Indeed, there are three possible displacement motions (sway or drift, surge and heave) that ships can undergo, as well as three angular motions (roll, yaw and pitch). Some authors have experimentally studied roll damping to visualize local flows, see for instance, [9, 5]. Meanwhile, researchers investigated mathematical modeling of roll damping. In this work we consider a model given by a second-order ordinary nonlinear differential equation with cubic damping moment and without forcing, namely

$$\ddot{x} + (2\mu + \delta x^2)\dot{x} + \omega^2 x - \frac{1}{a^2}x^3 = 0, \tag{1}$$

where the dot indicates differentiation with respect to the time variable t, ω is the dimensionless angular frequency, $-1/a^2$ is the strength of nonlinearity coefficient, μ is the dimensionless damping coefficient, and δ is the quadratic viscous damping coefficient. For more studies related with this equation, including the importance of its study, we refer the readers, in among, to [3, 5]. Particularly, in these works the authors used ultraspherical wavelets to obtain a numerical solution of equation (1).

2010 *Mathematics Subject Classification.* 34C05 (34C14).

Key words and phrases. Poincaré compactifcation, global phase portrarit.