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# Tangential Trapezoid Central Configurations

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**Abstract**—A tangential trapezoid, also called a circumscribed trapezoid, is a trapezoid whose four sides are all tangent to a circle within the trapezoid: the in-circle or inscribed circle. In this paper we classify all planar four-body central configurations, where the four bodies are at the vertices of a tangential trapezoid.

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## 1. INTRODUCTION

The classical $n$-body problem concerns the study of the dynamics of $n$ particles interacting among themselves by their mutual attraction according to Newtonian gravity.

Let $x_i \in \mathbb{R}^d (i = 1, \dots, n)$ denote the position vector of the $i$-body, and let $m_i \in \mathbb{R}^+ (i = 1, \dots, n)$ denote the mass of the $i$-body. $\mathbb{R}^d$ is the Euclidean space ($d = 2$ or 3). By Newton's law of motion and Newton's gravitational law the equations of motion of the $n$-body problem are governed by

$$\ddot{x}_i = -\sum_{j=1, j\neq i}^{n} \frac{m_j(x_i - x_j)}{r_{ij}^3}, \quad 1 \leqslant i \leqslant n,$$

where $r_{ij} = |x_i - x_j|$ is the mutual Euclidean distance between the $i$-body and the $j$-body. Here we take the gravitational constant $G = 1$.

The vector $x = (x_1, \dots, x_n) \in (\mathbb{R}^d)^n$ is called the *configuration* of the system. Define $\delta(x)$ as the dimension of a configuration $x$, i. e., the dimension of the smallest affine space of $\mathbb{R}^d$ containing all of the points $x_i$. Configurations with $\delta(x) = 1, 2, 3$ are called collinear, planar and spacial, respectively.

When $n = 2$, the $n$-body problem has been completely solved. However, for the $n$-body problem for $n \geqslant 3$ the complete solution remains open.

Let

$$M = m_1 + \dots + m_n, \quad c = \frac{m_1 x_1 + \dots + m_n x_n}{M}$$

be the total mass and the center of masses of the $n$ bodies, respectively.

A *configuration* $x$ is called a *central configuration* if the acceleration vectors of the $n$ bodies are proportional to their positions with respect to the center of masses with the same constant of proportionality, i. e.,

$$\sum_{j=1, j\neq i}^{n} \frac{m_j(x_j - x_i)}{r_{ij}^3} = \lambda(x_i - c), \quad 1 \leqslant i \leqslant n, \tag{1.1}$$

where $\lambda$ is the constant of proportionality.

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