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Asymptotic Development of an Integral Operator and Boundedness of the Criticality of Potential Centers

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Abstract
We study the asymptotic development at infinity of an integral operator. We use this development to give sufficient conditions to upper bound the number of critical periodic orbits that bifurcate from the outer boundary of the period function of planar potential centers. We apply the main results to two different families: the power-like potential family $\ddot{x} = x^q - x^p$, $p, q \in \mathbb{R}$, $p > q$; and the family of dehomogenized Loud's centers.

Keywords Center · Period function · Critical periodic orbit · Bifurcation · Criticality

Mathematics Subject Classification 34C07 · 34C23 · 34C25

1 Introduction

Consider a continuous family of planar potential systems

$$\dot{x} = -y, \ \ \dot{y} = V_\mu'(x), \tag{1}$$

where $\mu \in \Lambda$ is a parameter, Λ is an open subset of $\mathbb{R}^d$, $d \geqslant 1$, and V_μ is an analytic function defined in an open interval $I_\mu \subset \mathbb{R}$ containing $x = 0$. In the case when $V_\mu(0) = V_\mu'(0) = 0$ and $V_\mu''(0) > 0$ Eq. (1) has a non-degenerate center at the origin for each value of the parameter and so the point $(0, 0)$ has a punctured neighbourhood that is entirely foliated by periodic orbits surrounding it. The largest neighbourhood with this property is the period annulus of the center and we shall denote it by $\mathscr{P}_\mu$. If we consider the embedding of $\mathscr{P}_\mu$ into $\mathbb{RP}^2$, its boundary, namely $\partial\mathscr{P}_\mu$, is divided in two connected components: the origin itself, which is called the inner boundary of the period annulus, and the outer boundary of the period annulus defined by $\Pi_\mu := \partial\mathscr{P}_\mu \backslash \{(0, 0)\}$. When the center is a potential oscillator the natural parametrization of the closed orbits inside the period annulus is given by the energy level of the Hamiltonian $H(x, y; \mu) = \frac{1}{2}y^2 + V_\mu(x)$. Since $V_\mu(0) = 0$ by convention, we

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