PERIODIC SOLUTIONS FOR TWO CLASSES OF DUFFING DIFFERENTIAL EQUATIONS VIA AVERAGING THEORY

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Abstract. The aim of this paper is to provide sufficient conditions for the existence of periodic solutions in two classes of Duffing differential equation. The first class is

$$\ddot{x} + \varepsilon p(t)\dot{x} + (1 + \varepsilon q(t))x = \varepsilon f(t, x) + \varepsilon c(t),$$

where $p(t), q(t), f(t, x)$ and $c(t)$ are 2π–periodic functions in the variable t, ε is a small parameter and $x \in \mathbb{R}$. The second class is

$$\ddot{x} + (1 + \varepsilon\mu(t))\dot{x} + \varepsilon \sum_{i=0}^{n} \rho_{2i+1}(t)x^{2i+1} = \varepsilon f(t, x)$$

where $\mu(t), \rho_{2i+1}(t)$ with $i = 0, ..., n$ and $f(t, x)$ are C^2 functions $T-$periodic in the variable t, ε is a small parameter and $x \in \mathbb{R}$. Our results are proved using the averaging theory. Moreover we provide some applications.

1. Introduction and statement of the main results

These last years many results have been published on the periodic solutions of different classes of Duffing differential equations. These results are on the existence of periodic solutions, in their multiplicity, in their kind of stability, in their bifurcations,... see for instance [1, 3, 2, 4, 5, 6, 7, 8, 10, 12, 13, 15, 18, 19, 21].

Many authors discussed the existence of periodic solutions for the class of Duffing differential equations in $\mathbb{R}$ of the form

$$\ddot{x} + p(t)\dot{x} + q(t)x = f(t, x) + c(t), \tag{1}$$

where $p(t), q(t), f(t, x)$ and $c(t)$ are 2π–periodic functions in the variable t, $x \in \mathbb{R}$, and the dot denotes derivative with respect to the variable t. They have studied these equations under different additional conditions applying mainly the Schauder's and Krasnoselskii's Fixed Point Theorems, see for instance [4, 5, 6, 7, 13, 15, 18, 19, 21].

In this paper we provide sufficient conditions for the existence of periodic solutions for the class of Duffing differential equations in $\mathbb{R}$ of the form

$$\ddot{x} + \varepsilon p(t)\dot{x} + (1 + \varepsilon q(t))x = \varepsilon f(t, x) + \varepsilon c(t), \tag{2}$$

where $p(t)$, $q(t)$, $f(t, x)$ and $c(t)$ are 2π–periodic functions in the variable t, ε is a small parameter, and $x \in \mathbb{R}$. But here the results are obtained using the averaging theory.

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