Non-bifurcation of critical periods from semi-hyperbolic polycycles of quadratic centers

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Abstract. In this paper we consider the unfolding of saddle-node

$$X = \frac{1}{xU_a(x,y)}\Big(x(x^\mu - \varepsilon)\partial_x - V_a(x)y\partial_y\Big),$$

parametrized by (ε, a) with $\varepsilon \approx 0$ and a in an open subset A of $\mathbb{R}^\alpha$, and we study the Dulac time $\mathcal{T}(s;\varepsilon,a)$ of one of its hyperbolic sectors. We prove (Theorem A) that the derivative $\partial_s\mathcal{T}(s;\varepsilon,a)$ tends to $-\infty$ as $(s,\varepsilon) \to (0^+, 0)$ uniformly on compact subsets of A. This result is addressed to study the bifurcation of critical periods in the Loud's family of quadratic centers. In this regard we show (Theorem B) that no bifurcation occurs from certain semi-hyperbolic polycycles.

1 Introduction and main results

The present paper deals with planar polynomial ordinary differential systems and we study the qualitative properties of the period function of centers. A singular point of a planar differential system is a *center* if it has a punctured neighbourhood that consists entirely of periodic orbits surrounding it. The largest neighbourhood with this property is called the *period annulus* of the center and we denote it by $\mathscr{P}$. The *period function* assigns to each periodic orbit in $\mathscr{P}$ its period. If the period function is constant then the center is called *isochronous*. The study of the period function is a nontrivial problem and questions related to its behaviour have been extensively studied. Let us quote, for instance, the problems of isochronicity (see [4, 5, 25]), monotonicity (see [1, 24, 27]) or bifurcation of critical periods (see [2, 12, 13]). Aside from the intrinsic interest of these problems, the study of the period function is also important in the analysis of nonlinear boundary value problems and in perturbation theory. Indeed, for instance, under the condition of non-criticality of the period function, zeros of appropriate Melnikov functions guarantee the persistence of subharmonic periodic orbits of a Hamiltonian system after a small periodic non-autonomous perturbation (see [8, 15]). Most of the work on planar polynomial differential

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