Bifurcations of zeros in translated families of functions and applications

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Abstract. In this paper we study the creation of zeros in a certain type of families of functions. The families studied are given by the difference of two basic functions with a translation made in the argument of one of these functions. The problem is motivated by applications in the 16-th Hilbert problem and its ramifications. Here, we apply the results obtained to the study of bifurcations of critical periods in the Loud family of quadratic centers.

1 Introduction and main results

This paper is motivated by the study of bifurcations of critical points of the period function in a neighborhood of a polycycle. A key problem in theses studies is the breaking of separatrices of the polycycle. It appears also in the study of limit cycles corresponding to fixed points of the Poincaré return map of a family of planar vector fields. Contrary to the situation in the study of limit cycles, here by breaking a polycycle it is replaced by a polycycle with less vertices. The simplest situation is when a polycycle with two vertices is broken and a saddle loop polycycle is created.

The cyclicity (i.e. number of limit cycles appearing by perturbation) of hyperbolic polycycles has been extensively studied in [1, 20, 24, 16, 17, 18, 15, 4] among others.

On the other hand, in our study of critical points of the period function of Loud systems [9, 6, 7, 10, 13] we gave a conjectural bifurcation diagram. We could not prove it in full generality due in part to some phenomena of breaking of separatrices of the polycycle bounding the period annulus. Here we deal with this problem. The simplest setting is the breaking of one separatrix (or two separatrices in the presence of symmetry).

This problem leads to the following type of equation

$$\Delta(s;\varepsilon,\mu) := F_1(s;\mu) - F_2(s+\varepsilon;\mu) = 0, \tag{1}$$

where $s = 0$ corresponds to the polycycle, $s \geq 0$ parametrizes the monodromic region, $\varepsilon \approx 0$ is the parameter controlling the breaking of the separatrix and μ regroups all other parameters of the family, which we study in a neighborhood of a parameter value μ_0.

We study the family of functions $\Delta_\nu(s) = \Delta(s,\varepsilon;\mu)$, for the parameter $\nu = (\varepsilon,\mu)$ in a neighborhood of $\nu_0 = (0,\mu_0)$ and $s \geq 0$ close to 0.