

A SURVEY ON ALGEBRAIC AND EXPLICIT NON-ALGEBRAIC LIMIT CYCLES IN PLANAR DIFFERENTIAL SYSTEMS

JAUME LLIBRE¹ AND XIANG ZHANG²

ABSTRACT. In the qualitative theory of differential equations in the plane one of the most difficult objects to study is the existence of limit cycles. There are many papers dedicated to this subject. Here we will present a survey mainly dedicated to the algebraic and explicit non-algebraic limit cycles of the polynomial differential systems in \mathbb{R}^2 and of the discontinuous piecewise differential systems in \mathbb{R}^2 formed by two linear differential systems separated by a straight line. For this class of discontinuous piecewise differential systems the study of their algebraic and explicit non-algebraic limit cycles just is starting. Here we provide the first explicit non-algebraic limit cycle for the discontinuous piecewise linear differential systems. Additionally we recall seven open questions related with these types of limit cycles.

1. INTRODUCTION

We start by recalling the definition of the two classes of differential systems whose algebraic and explicit non-algebraic limit cycles we will study.

Let $P(x, y)$ and $Q(x, y)$ be real polynomials in the variables x and y . Then the differential system

$$(1) \quad \begin{aligned} \dot{x} &= P(x, y), \\ \dot{y} &= Q(x, y), \end{aligned}$$

where as usual the dot denotes the derivative with respect to the independent variable t , is a *polynomial differential system*. The maximum of the degrees of the polynomials P and Q is the *degree* of the polynomial differential system (1).

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