CUBIC POLYNOMIAL HAMILTONIAN SYSTEMS WITH DEGENERATE CENTERS

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ABSTRACT. In this note we solve the following two questions; *Are there degenerate centers inside the class of cubic polynomial Hamiltonian systems in* $\mathbb{R}^2$? If the answer is positive, *what are the phase portraits of the cubic polynomial Hamiltonian systems having a degenerate center*?

1. Introduction and statement of the main results

A *center* is an equilibrium point p of a differential system in the plane $\mathbb{R}^2$ if there is a neighborhood U of p such that p is the unique equilibrium in U and $U \setminus \{p\}$ is filled by periodic orbits. The center p is *global* if $\mathbb{R}^2 \setminus \{p\}$ is filled by periodic orbits. The notion of center goes back to Poincaré [14] and Dulac [8].

Using the linear part of a differential system in a center, the center can be classified into three types. More precisely, after moving the center to the origin of coordinates and making a linear change of variables, and a rescaling of the time variable (if they are necessary), the polynomial differential system in $\mathbb{R}^2$ can be written in one of the following three forms:

$$\begin{aligned} \dot{x} &= -y + X_2(x, y), \\ \dot{y} &= x + Y_2(x, y), \end{aligned}$$

which is called a *linear type center* or *elementary center*;

$$\begin{aligned} \dot{x} &= y + X_2(x, y), \\ \dot{y} &= Y_2(x, y), \end{aligned}$$

which is called a *nilpotent center*,

$$\begin{aligned} \dot{x} &= X_2(x, y), \\ \dot{y} &= Y_2(x, y), \end{aligned}$$

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