PERIODIC SOLUTIONS AND THEIR STABILITY FOR SOME PERTURBED HAMILTONIAN SYSTEMS

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Abstract. We deal with non–autonomous Hamiltonian systems of one degree of freedom. For such differential systems we present some results for computing analytically some of their periodic solutions, together with their type of stability. The tool for proving these results is the averaging theory. We present some applications of these results.

1. Introduction and main results

We consider the following perturbed first order differential systems

$$\frac{dp}{dt} = -\frac{\partial \mathcal{H}}{\partial q} + \varepsilon \mathcal{P}_1(q,p,t), \qquad \frac{dq}{dt} = \frac{\partial \mathcal{H}}{\partial p} + \varepsilon \mathcal{P}_2(q,p,t), \tag{1}$$

where $\mathcal{H} = \mathcal{H}(q,p)$ is a Hamiltonian function defined in an open set U of $\mathbb{R}^2$, q is the position, p its associated momentum, the smooth functions $\mathcal{P}_i; U \times \mathbb{R}$ are $2\pi-$ periodic in t, and ε is a small parameter.

All the lemmas, theorems and corollaries state in this section are proved in the next two sections. The theorems are proved using the averaging theory, see a summary of the part of this theory that we need for proving our results in the appendix. The averaging theory is one of the best tools for computing analytically periodic solutions of the differential equations, see for instance [2].

We assume that the Hamiltonian $\mathcal{H}$ can be expressed in action–angle variables (I,θ) as $\mathcal{H}(p,q) = \mathcal{H}_0(I)$. If the change of variable $(p,q) \mapsto (I,\theta)$ is given by $I = I(p,q)$ and $\theta = \theta(p,q)$, then we consider the functions

$$\begin{aligned} \mathcal{F}_1(I,\theta,t) &= \frac{\partial I}{\partial p}\mathcal{P}_1(q,p,t) + \frac{\partial I}{\partial q}\mathcal{P}_2(q,p,t), \\ \mathcal{F}_2(I,\theta,t) &= \frac{\partial \theta}{\partial p}\mathcal{P}_1(q,p,t) + \frac{\partial \theta}{\partial q}\mathcal{P}_2(q,p,t). \end{aligned} \tag{2}$$

The following result holds taking into account that the change of variables to action–angle variables is canonical.

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