DYNAMICS OF A CLASS OF 3–DIMENSIONAL LOTKA–VOLTERRA SYSTEMS

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Abstract. We characterize the dynamics of the Lotka–Volterra differential system

$$\dot{x} = x(ay - cz), \quad \dot{y} = y(bz - ax), \quad \dot{z} = z(cx - by),$$

where a, b, c are positive parameters. This nonlinear differential system modelizes the autocatalytic chemical reactions between three macromolecules.

1. Introduction and statement of the main results

The Lotka–Volterra differential systems appeared in 1910 at the work of A.J. Lotka [20] for modeling autocatalytic chemical reactions, and was also applied by himself in 1920 to the system

$$\frac{dx}{dt} = x(\alpha - \beta y), \quad \frac{dy}{dt} = y(-\gamma + \delta x),$$

for modeling the dynamics between a herbivorous animal and a plant species (see [21]), where y and x are respectively the numbers of predators and preys, and the positive real parameters α, β, γ and δ describe the interaction between the two species. Later on in 1926 Volterra [27] developed the model of Lotka for explaining the exchange of the fish catches between fish and predatory fish. In 1936 Kolmogorov [12] considered these systems and extended them to higher dimension.

Lotka–Volterra differential systems have been generalized for studying the dynamics between the competition of two or more species, and to model dynamical phenomena in many distinct areas, such as plasma physics [13], hydrodynamics [4], evolution of conflicting species in biology [10, 24], chemical reactions [9], ... The Lotka–Volterra systems have been analyzed from many disticnt points of view, see [1, 3, 5, 14, 16, 17, 18, 23].

In biochemistry the pioneering work of Wyman [28] modelizing the autocatalytic chemical reactions, see also Di Cera et al. [7], has had many applications as circadian clocks [25], enzyme kinetics [28], and genetic networks [2, 6], ... When the law of mass conservation is considered it was proved in [7] that the autocatalytic chemical reactions

2000 *Mathematics Subject Classification.* 34C05, 34C23, 34C25, 34C29.

Key words and phrases. Lotka–Volterra system, invariant, global dynamics, phase portrait.