TWIN QUADRATIC POLYNOMIAL VECTOR FIELDS IN THE SPACE $\mathbb{C}^3$

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Abstract. We want to address the question of whether a polynomial vector field in $\mathbb{C}^3$ is completely determined by its singularities and their spectra. We say that two distinct vector fields are *twin* if they have the same singular locus and the same spectra at each singularity. Our results provide conditions for the existence of such vector fields.

1. Introduction and statement of the results

Consider polynomial vector fields on the affine space $\mathbb{C}^3$. We denote by $\mathcal{P}$ the space of all polynomial vector fields

$$\chi = P(x,y,z)\frac{\partial}{\partial x} + Q(x,y,z)\frac{\partial}{\partial y} + R(x,y,z)\frac{\partial}{\partial z}$$

such that P, Q and R are quadratic. By Bezout's Theorem, a generic element of $\mathcal{P}$ has exactly eight isolated singularities. We denote by $\mathcal{P}_8$ the space of the vector fields in $\mathcal{P}$ that have eight isolated singularities. Since χ has the maximum number of singularities the determinant of the linear part of χ at each singular point is nonzero. So the eigenvalues at any singular point are nonzero, i.e. all singular points are non degenerate (see for more details [4]). The space $\mathcal{P}_8$ is endowed with a structure of a complex affine space identifying all the thirty coefficients of the polynomials P, Q and R with a point of $\mathbb{C}^{30}$. This topology in the set $\mathcal{P}_8$ is called the *topology of the coefficients*, and $\mathcal{P}_8$ is an open subset of $\mathcal{P}$.

We say that two vector fields χ and $\widehat{\chi}$ of $\mathcal{P}_8$ are *affine equivalent* if there exists an affine transformation T that maps χ into $\widehat{\chi}$ that is

$$\widehat{\chi}(x,y,z) = DT \cdot \chi(T^{-1}(x,y,z)).$$

2010 *Mathematics Subject Classification.* 34C05, 34C07, 34C08.
Key words and phrases. Quadratic polynomial vector fields, singularities, spectra, twin vector fields.