THE MARKUS-YAMABE CONJECTURE FOR DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS IN $\mathbb{R}^n$ SEPARATED BY A CONIC$\times\mathbb{R}^{n-2}$

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ABSTRACT. In 1960 Markus and Yamabe made the conjecture that if a C^1 differential system $\dot{x} = F(x)$ in $\mathbb{R}^n$ has a unique equilibrium point and $DF(x)$ is Hurwitz for all $x \in \mathbb{R}^n$, then the equilibrium point is a global attractor. This conjecture was completely solved in 1997 and it turned out to be true in $\mathbb{R}^2$ and false in $\mathbb{R}^n$ for all $n \geq 3$.

In [17] the authors extended the Markus–Yamabe conjecture to continuous and discontinuous piecewise linear differential systems in $\mathbb{R}^n$ separated by a hyperplane, they proved for the continuous systems that the extended conjecture is true in $\mathbb{R}^2$ and false in $\mathbb{R}^n$ for all $n \geq 3$, but for discontinuous systems they proved that the conjecture is false in $\mathbb{R}^n$ for all $n \geq 2$.

In this paper first we show that there are no continuous piecewise linear differential systems separated by a conic$\times\mathbb{R}^{n-2}$ except the linear differential systems in $\mathbb{R}^n$. And after we prove that the extended Markus–Yamabe conjecture to discontinuous piecewise linear differential systems in $\mathbb{R}^n$ separated by a conic$\times\mathbb{R}^{n-2}$ is false in $\mathbb{R}^n$ for all $n \geq 2$.

1. Introduction and statement of the results

Consider a C^1 differential system $\dot{x} = F(x)$ defined in $\mathbb{R}^n$ and having an equilibrium point at the origin of coordinates. If $DF(0)$ is Hurwitz (i.e. the eigenvalues of $DF(0)$ have negative real part), then by the Hartman-Grobman Theorem [11, 14] the origin is locally asymptotically stable. A natural question arises: which are the additional hypotheses that one may add to the function F in order that the origin is a global attractor.

Markus and Yamabe in 1960 (see [18]) made the following conjecture: If we have a C^1 differential system $\dot{x} = F(x)$ defined in $\mathbb{R}^n$ such that $DF(x)$ is Hurwitz for all $x \in \mathbb{R}^n$, and having a unique equilibrium point at the origin of coordinates, then the origin is a global attractor.

This conjecture follows easily when $n = 1$. This conjecture when $n = 2$ was proved independently by Gutierrez [12, 13] in 1993 and by Fessler [6, 7] in 1995. A simpler proof was then given by Glutsyuk in [9, 10]. The counterexample to Markus-Yamabe conjecture for $n > 3$ was given by Bernat and Llibre in [3] and the counterexample for $n \geq 3$ was given by Cima, van den Essen, Gasull, Hubbers and

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