PHASE PORTRAITS OF UNIFORM ISOCHRONOUS CENTERS WITH HOMOGENEOUS NONLINEARITIES

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ABSTRACT. We classify the phase portraits in the Poincaré disc of the differential equations of the form $x' = -y + xf(x, y)$, $\dot{y} = x + yf(x, y)$ where $f(x, y)$ is a homogeneous polynomial of degree $n - 1$, and f has only simple zeroes when $n = 2, 3, 4, 5$. We also provide some general results on these uniform isochronous centers for all $n \geq 2$.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The first investigation in isochronicity goes back to Huygens in [6] with the study of the cycloidal pendulum in the XVII century. Nowadays isochronicity appears in many physical problems and it is closely related with the existence and uniqueness of solutions for certain bifurcation problems or boundary value problems (see for instance [8] and the references therein). In the last decade the study of the isochronicity has been grown specially in the case of polynomial differential systems due to the appearance of powerful methods of computational analysis, see for instance [1, 3, 4, 14] to cite just few of them.

A *polynomial differential system of degree* n is a differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

with P and Q polynomials such that the maximum of their degrees is n. We denote by $\chi = (P, Q)$ the polynomial vector field associated to system (1).

Let $p \in \mathbb{R}^2$ be a center of χ. We say that p is an *isochronous center* of χ if it is a center with a neighborhood surrounded by periodic orbits with the same period. We say that p is a *uniform isochronous center* of χ if the system associated to χ can be written in polar coordinates in the form

$$\dot{r} = G(\theta, r), \quad \dot{\theta} = \kappa, \quad \kappa \in \mathbb{R} \setminus \{0\}.$$

We say that p is a *global center* if its period annulus is $\mathbb{R}^2$. We recall that a *period annulus* of a center p is the maximum neighborhood $U \subset \{p\}$ of the center p filled up with periodic orbits.

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