RATIONAL LIMIT CYCLES ON ABEL EQUATIONS

JAUME LLIBRE[1] AND CLÀUDIA VALLS[2]

ABSTRACT. In this paper we deal with Abel equations $dy/dx = A(x)y^2 + B(x)y^3$, where $A(x)$ and $B(x)$ are real polynomials. We prove that these Abel equations can have at most three rational limit cycles and we characterize when this happens. Moreover, we provide examples of these Abel equations with three nontrivial rational limit cycles. We also prove that in this case the limit cycles cannot be hyperbolic.

1. Introduction and statement of the results

We study the Abel equations

$$\frac{dy}{dx} = A(x)y^2 + B(x)y^3, \tag{1}$$

where x, y are real variables and $A(x)$ and $B(x)$ are polynomials. The limit cycles of these equations have been intensively investigated mainly when the functions $A(x)$ and $B(x)$ are periodic (see for instance [1, 2, 3, 4, 5, 6, 7, 9, 12, 13, 15, 16, 17, 18, 19, 21, 22, 23, 24]), and also when $A(x)$ and $B(x)$ are polynomial (see for instance [8, 10, 11, 14, 20]). Here we are interested in the rational limit cycles of equation (1) when the functions $A(x)$ and $B(x)$ are polynomial.

A *periodic solution* of equation (1) is a solution $y(x)$ defined in the closed interval $[0, 1]$ such that $y(0) = y(1)$.

We say that a *limit cycle* is a periodic solution isolated in the set of periodic solutions of a differential equation (1). Without loss of generality we will assume that the period is 1.

The limit cycle is called a *polynomial limit cycle* if the periodic solution $y(x)$ is a polynomial in the variable x. In particular the authors of [14] proved that any polynomial limit cycle of system (1) is of the form $y = c$ with $c \in \mathbb{R}$, and that if a polynomial limit cycle exists with $c \neq 0$, then no other polynomial limit cycles can exist.

In this paper we want to consider the existence of *rational limit cycles* for system (1), i.e. we want to consider limit cycles of the form $y(x) = q(x)/p(x)$

2010 *Mathematics Subject Classification.* 34C05, 34C07, 34C08.
Key words and phrases. Algebraic limit cycles, Rational limit cycles, Abel equations, Hyperbolic limit cycles.