ON THE CONFIGURATIONS OF THE SINGULAR POINTS AND THEIR TOPOLOGICAL INDICES FOR THE SPATIAL QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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Abstract. Using the Euler-Jacobi formula there is a relation between the singular points of a polynomial vector field and their topological indices. Using this formula we obtain the configuration of the singular points together with their topological indices for the polynomial differential systems $\dot{x} = P(x, y, z)$, $\dot{y} = Q(x, y, z)$, $\dot{z} = R(x, y, z)$ with degrees of P, Q and R equal to two when these systems have the maximum number of isolated singular points, i.e., 8 singular points. In other words we extend the well-known Berlinskii's Theorem for quadratic polynomial differential systems in the plane to the space.

1. Introduction and statement of the main results

Consider in $\mathbb{R}^3$ the polynomial differential systems

$$\dot{x} = P(x, y, z), \quad \dot{y} = Q(x, y, z), \quad \dot{z} = R(x, y, z), \tag{1}$$

where $P(x, y, z)$, $Q(x, y, z)$ and $R(x, y, z)$ are real polynomials of degrees 2, called spatial quadratic polynomial differential system, or simply *quadratic systems* in what follows.

The motivation of our paper comes from the fact that for the planar quadratic polynomial differential systems the characterization of all configurations of the (topological) indices of the singular points of these systems having four singular points is the well-known Berlinskii's Theorem proved in [2, 7] and reproved in [5] using the Euler-Jacobi formula. More precisely, the Berlinskii's Theorem can be stated as follows: *Assume that a real planar quadratic polynomial differential system has exactly four real singular points. In this case if the quadrilater formed by these points is convex, then two opposite singular points are anti-saddles (i.e. nodes, foci or centers) and the other two are saddles. If this quadrilater is not convex, then either the three exterior vertices are saddles and the interior vertex is an anti-saddle or the exterior vertices are anti-saddles and the interior vertex is a saddle.*

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