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Global qualitative dynamics of the Brusselator system

Jaume Llibre[a,*], Clàudia Valls[b]

[a] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona,Catalonia, Spain*
[b] *Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1049–001, Lisboa, Portugal*

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Abstract

We study the dynamics of the Brusselator model by analysing the flow of this differential system in the Poincaré disc.

Keywords: Brusselator system; Nilpotent saddle; Phase portrait; Poincaré compactification; Global attractor

1. Introduction and statement of the results

We consider the differential system

$$\begin{aligned} \dot{x} &= a - (b + c)x + x^2 y, \\ \dot{y} &= bx - x^2 y, \end{aligned} \tag{1}$$

called the *Brusselator model* where x and y are the dimensionless concentration of some species and the parameters a, b, c are all positive. As usual the dot denotes derivative with respect to the time t.

Such differential system appears in several branches of the sciences, mainly in chemistry because it exhibits kinetics of model trimolecular irreversible reactions (see [2,4,6,7,13] for details). The first integrals of system (1) in function of its parameters were studied in [8].

Here we study the qualitative behaviour of all the solutions of system (1), by describing its phase portraits in the Poincaré disc in function of its parameters. See the Appendix for the definitions and the basic results that we use, and in particular for the definition of the Poincaré disc. Roughly speaking the Poincaré disc is the 2-dimensional closed unit disc centred at the origin of coordinates, its interior is identified with $\mathbb{R}^2$ and its boundary (the circle $\mathbb{S}^1$) is identified with the infinity of $\mathbb{R}^2$, i.e. in $\mathbb{R}^2$ we can go to or come from infinity in as many directions as points has the circle.

We say that two vector fields on the Poincaré disc are *topologically equivalent* if there exists a homeomorphism of the Poincaré disc which sends orbits to orbits preserving or reversing their orientation.

* Corresponding author.
E-mail addresses: jllibre@mat.uab.cat (J. Llibre), cvalls@math.ist.utl.pt (C. Valls).

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