ON THE SINGULARITIES OF THE PLANAR CUBIC POLYNOMIAL DIFFERENTIAL SYSTEMS AND THE EULER JACOBI FORMULA

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ABSTRACT. Using the Euler-Jacobi formula we obtain an algebraic relation between the singular points of a polynomial vector field and their topological indices. Using this formula we obtain the configuration of the singular points together with their topological indices for the planar cubic polynomial differential systems when these systems have nine finite singular points.

1. Introduction and statement of the main results

Consider in $\mathbb{R}^2$ the polynomial differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are real polynomials of degree 3, called a planar cubic polynomial differential system, or simply *cubic system*.

The motivation of our paper comes from the fact that for the planar quadratic polynomial differential systems the characterization of all configurations of the indices of the singular points of all quadratic differential systems that have four singular points is the well-known Berlinskii's Theorem proved in [2, 6], and reproved in [4] using the Euler-Jacobi formula. We say that a quadrilateral is *convex* if any vertex of it is contained in the convex hull of the other three vertices, otherwise the quadrilateral is called *concave*. Then the Berlinskii's Theorem can be stated as follows. *Assume that a real quadratic differential system has exactly four real singular points. In this case if the quadrilateral formed by these points is convex, then two opposite singular points are anti-saddles (i.e. nodes, foci or centers) and the other two are saddles. If this quadrilateral is concave, then either the three exterior vertices are saddles and the interior vertex is an anti-saddle, or the exterior vertices are anti-saddles and the interior vertex is a saddle.*

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