CONFIGURATIONS OF THE TOPOLOGICAL INDICES OF THE PLANAR POLYNOMIAL DIFFERENTIAL SYSTEMS OF DEGREE $(1, m)$

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ABSTRACT. Using the Euler-Jacobi formula there is a relation between the singular points of a polynomial vector field and their topological indices. Using this formula we obtain the configurations of the topological indices of the singular points for the polynomial differential systems $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$ with degree of P equal to 1 and degree of Q equal to $m \geq 1$ when these systems have m finite singular points.

1. Introduction and statement of the main results

Consider in $\mathbb{R}^2$ the polynomial differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are real polynomials of degrees 1 and m, respectively, or simply of degree $(1, m)$.

The motivation of our paper comes from the fact that for the planar quadratic polynomial differential systems (i.e. the ones of degree $(2, 2)$) the characterization of all configurations of the indices of the singular points of all systems that have four singular points is the well-known Berlinskii's Theorem proved in [2, 5] and reproved in [3] using the Euler-Jacobi formula. More precisely, the Berlinskii's Theorem can be stated as follows: *Assume that a real quadratic system has exactly four real singular points. In this case if the quadrilateral formed by these points is convex, then two opposite singular points are anti-saddles (i.e. nodes, foci or centers) and the other two are saddles. If this quadrilateral is not convex, then either the three exterior vertices are saddles and the interior vertex is an anti-saddle or the exterior vertices are anti-saddles and the interior vertex is a saddle.*

We want to extend the Berlinskii's Theorem from degree $(2, 2)$ to degree $(1, m)$ for all $m \geq 2$, i.e., we shall obtain all configurations of the singular points together with their topological indices for the polynomial differential systems (1) when these systems have m finite singular points.

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