A SUFFICIENT CONDITION FOR THE REAL JACOBIAN CONJECTURE IN $\mathbb{R}^2$

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ABSTRACT. Let $F = (f, g)\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that $\det DF(x, y)$ is different from zero for all $(x, y) \in \mathbb{R}^2$. We provide some new sufficient conditions for the injectivity of F. The proofs are based on the qualitative theory of differential equations.

1. Introduction and statement of the main result

Let $F = (f, g)\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a smooth map such that $\det DF(x, y)$ is different from zero for all $(x, y) \in \mathbb{R}^2$. By the Inverse Function Theorem, it is clear that F is a local diffeomorphism, but it is not always injective. There are very general well known additional conditions to ensure that F is a global diffeomorphism, see for instance [10, 14, 17].

If F is a polynomial map, the statement that F is injective is known as the *real Jacobian conjecture*. This conjecture is false, because Pinchuk constructed, in [16], a non-injective polynomial map with nonvanishing Jacobian determinant. Thus it is natural to ask for additional conditions in order that this conjecture holds. In [5, 6], for instance, it was shown that for the injectivity of F it is enough to assume that the degree of f is less than or equal to 4. If we assume that $\det DF(x)$ is a constant different from zero, then to know if F is injective is an open problem largely known as the *Jacobian conjecture* (see [1] and [12] for details and for surveys on the Jacobian conjecture and related problems). In [2] the authors provide a sufficient condition for the validity of the real Jacobian conjecture. More precisely they proved the following theorem.

Theorem 1. *Let $F = (f, g)\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that* $\det DF$ *is nowhere zero. If the higher homogeneous terms of the*

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