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THE ZERO-HOPF BIFURCATIONS OF A FOUR-DIMENSIONAL HYPERCHAOTIC SYSTEM

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ABSTRACT. We consider the four-dimensional hyperchaotic system $\dot{x} = a(y - x)$, $\dot{y} = bx + u - y - xz$, $\dot{z} = xy - cz$, $\dot{u} = -du - jx + exz$, where a, b, c, d, j, e are real parameters. This system extend the famous Lorenz system to dimension four and was introduced in the paper of the Internat. J. Bifur. Chaos Appl. Sci. Engrg., **27** (2017), 1750021. We characterize the values of the parameters for which its equilibrium points are zero-Hopf points. Using the averaging theory we obtain sufficient conditions for the existence of periodic orbits bifurcating from these zero-Hopf equilibria, and give some examples to illustrate the conclusions. Moreover the stability conditions of these periodic orbits are given using the Routh-Hurwitz criterion.

1. INTRODUCTION

Chaos phenomenon is a complex dynamic behavior in nonlinear dynamical system, which appears in nature widely. In 1963, the meteorologist Edward Lorenz [24] was the first to introduce the mathematical and physical chaotic model in $\mathbb{R}^3$, which is known as the Lorenz system. The Lorenz system planted the seed in the chaos science. This system plays an important effect in other areas as in the modeling of lasers [11] and dynamos [12]. As one of the simplest models presenting chaos, the Lorenz system exhibits a rich range of dynamical properties, and it has been researched from different points of view, such as positively invariant [17], integrability [22, 16, 14], global dynamics [34, 4, 26] and bifurcation [3, 32]. After that Lorenz system, mathematicians and physicists from physical or purely abstract mathematical point of view proposed various polynomial differential systems in $\mathbb{R}^3$, whose trajectories exhibit chaotic dynamics of the Lorenz system type. As examples, one can refer to Rikitake system [20], Sprott A system [1], Shimizu-Morioka system [13], etc.

Nowadays three-dimensional nonlinear systems cannot provide adequate description of many phenomena in neural networks, social sciences and engineering, etc. To better describe the real world, we often necessitate to introduce high-dimensional (dimension at least four) nonlinear systems. Recently the hyperchaotic system has become a focus research subject, see [6, 9, 30, 31, 5, 35, 27, 7] and the references therein. The concept of hyperchaos was given by Rössler in [29]. The precise definition of *hyperchaotic system* is: (i) at least four-dimensional autonomous differential system, (ii) a dissipative structure, and (iii) at least two unstable directions, of

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2020 Mathematics Subject Classification. Primary 34C23, Secondary 34C25, 34C29.

Key words and phrases. Hyperchaotic system, zero-Hopf bifurcation, periodic orbits, averaging theory.