LIMIT CYCLES IN FILIPPOV SYSTEMS HAVING A CIRCLE AS SWITCHING MANIFOLD

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ABSTRACT. It is known that planar discontinuous piecewise linear differential systems separated by a straight line have no limit cycles when both linear differential systems are centers.

Here we study the limit cycles of the planar discontinuous piecewise linear differential systems separated by a circle, when both linear differential systems are centers. Our main results show that such discontinuous piecewise differential systems can have 0, 1 or 2 limit cycles, but no more limit cycles than 2.

1. Introduction and statement of the main result

1.1. **Historical facts.** The problem of existence of limit cycles has been extensively treated in the literature since the early days of celestial mechanics. More recently, much work has been done on the rigorous mathematical foundation of nonsmooth dynamical systems problems, in particular, in the search of typical minimal sets that there are no counterparts in the smooth universe. It is worth to mention that, some existing smooth techniques are useful in solving many nonsmooth problems.

Some of the orbits of planar differential systems are difficult to study, this is the case of the limit cycles. Recall that a *limit cycle* of a differential system (S) is a periodic solution of (S) which is isolated in the set of all periodic solutions of (S). Concerning the nonsmooth universe and in the 2–dimensional case one can find many results on the existence of limit cycles when the switching set is an imbedded curve in $\mathbb{R}^2$, see [1–15, 18–29].

One of the main properties of smooth integrable systems in the plane $\mathbb{R}^2$ is that their periodic orbits usually appear in continuous one–parameter families, in contrast to the periodic orbits of piecewise nonsmooth integrable systems which typically are limit cycles, see [2–6, 18–28].

Andronov, Vitt and Khaikin [1] started the study of the discontinuous piecewise linear differential systems in the plane, mainly motivated for their applications to some mechanical problems. Recently the interest for this kind of differential systems increased due mainly to the fact that these differential systems model many processes appearing in mechanics, electronics, economy,... See for these applications the survey of Makarenkov and Lamb [27], and the books of Simpson [30] and of

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