PHASE PORTRAITS OF A SIR EPIDEMIC MODEL

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Abstract. In this paper we classify the phase portraits of a SIR epidemic dynamics model. Depending on the values of the parameters this model can exhibit seven different phase portraits. In particular, from a biological point of view we prove that the unique attractors of this model are one or two equilibrium points depending on the values of the parameters, and from the phase portraits follow the basins of attraction of these equilibria.

1. Introduction and statement of the main results

The study of a widespread occurrence of an infectious disease in a community at a particular time has great importance, through decades many models have emerged to give explication for the spread and reason of epidemic outbreaks. The SIR [8] is one of the very realistic models that gives a good explanation of the spread of infectious disease which is used to study many epidemics like cholera [12], malaria [1] and very recently the novel coronavirus (2019-nCoV or COVID-19) which has been a great worry around the globe, thus scholars have focused their energies in studying the behavior of such fatal disease [3, 5, 13, 14].

In the SIR infections disease model, the total population N is composed of three groups of individuals which are: $S(t)$ denote the number of members of a population susceptible to the disease at time t; $I(t)$ represents the number of infective members, and $R(t)$ denotes the number of members who have been removed from the population (see [2, 4, 6]). The SIRS [4] model is a SIR model, which allows recovered individuals to return to a susceptible state, in the case when this model has birth rate and death rate can be written as

$$
\begin{aligned}
\frac{dS}{d\tau} &= -IH(I,S) - r_2 S + r_1 R + B(N),\\
\frac{dI}{d\tau} &= IH(I,S) - (r_2+\nu)I,\\
\frac{dR}{d\tau} &= \nu I - (r_2+r_1)R,
\end{aligned}
\tag{1}
$$

where

- $H(I,S)$ is th nonlinear incidence rate concerning S and I;
- r_2 is the common natural death rate of the three groups (S, I and R);
- $B(N)$ denotes the birth rate and is a function depending on $N=S+I+R$;

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