ON A CLASS OF POLYNOMIAL DIFFERENTIAL SYSTEMS OF DEGREE 4: PHASE PORTRAITS AND LIMIT CYCLES

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Abstract. In this paper we characterize the phase portraits in the Poincaré disc of the class of polynomial differential systems of the form

$$\dot{x} = -y, \qquad \dot{y} = x + ax^4 + bx^2y^2 + cy^4,$$

with $a^2 + b^2 + c^2 \neq 0$, which are symmetric with respect to the x-axis. Such systems have a center at the origin of coordinates. Moreover using the averaging theory of five order we study the number of limit cycles which can bifurcate from the period annulus of this center when it is perturbed inside the class of all polynomial differential systems of degree 4.

1. Introduction and statement of the main results

By definition a *polynomial differential system* in $\mathbb{R}^2$ is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \qquad \frac{dy}{dt} = \dot{y} = Q(x, y), \tag{1}$$

where the dependent variables are x and y, and the independent one (the time) is denoted by t, and $P(x, y)$ and $Q(x, y)$ are polynomials in the variables x and y with real coefficients. We denote by $m = \max\{\deg P, \deg Q\}$ the *degree* of the polynomial differential system.

A singular point p of a polynomial differential system (1) is a *center* if there is a neighborhood U of p such that $U \setminus \{p\}$ is filled by periodic orbits.

A *limit cycle* of a polynomial differential system (1) is a periodic orbit isolated in the set of all periodic orbits of system (1).

The *phase portrait* of a differential system is the decomposition of its domain of definition as union of all its oriented orbits. The phase portraits of the polynomial differential systems are drawn in the Poincaré disc which, roughly speaking, is the closed disc centered at the origin of coordinates with radius one, the interior of this disc is diffeomorphic to $\mathbb{R}^2$ and its boundary $\mathbb{S}^1$ corresponds to the infinity of $\mathbb{R}^2$, each point of $\mathbb{S}^1$ provides a direction for going or coming from infinity. For more details see Chapter 5 of [4], we shall use the conditions and notations introduced in that chapter.

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