Periodic orbits and equilibria for a seventh-order generalized Hénon-Heiles Hamiltonian system

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Abstract

In this paper we study analytically the existence of two families of periodic orbits using the averaging theory of second order, and the finite and infinite equilibria of a generalized Hénon-Heiles Hamiltonian system which includes the classical Hénon-Heiles Hamiltonian. Moreover we show that this generalized Hénon-Heiles Hamiltonian system is not C^1 integrable in the sense of Liouville–Arnol'd, i.e. it has not a second C^1 first integral independent with the Hamiltonian. The techniques that we use for obtaining analytically the periodic orbits and the non C^1 Liouville-Arnol'd integrability, can be applied to Hamiltonian systems with an arbitrary number of degrees of freedom.

Keywords: eneralized Hénon-Heiles potential – Finite equilibria – Infinite equilibria

1. Introduction and statement of results

The classical Hénon-Heiles Hamiltonian consist of a two dimensional harmonic potential plus two cubic terms,i.e.

$$H = \frac{1}{2}(p_x^2 + p_y^2 + x^2 + y^2) + x^2 y - \frac{y^3}{3}.$$

This Hamiltonian was introduced in 1964, it is a model for studying the existence of a third integral of motion of a star in an rotating meridian plane of a galaxy in the neighborhood of a circular orbit [1].

The generalized Hénon-Heiles Hamiltonian system here studied is

$$\begin{aligned} H_\varepsilon = \ & \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(x^2 + y^2) + x^2 y - \frac{y^3}{3} + \varepsilon\Big(x^6 y + x^4 y^3 + x^4 y \\ & + x^2 y^5 + x^2 y^3 - \frac{y^7}{7} - \frac{y^5}{5} + \frac{1}{4}(x^2+y^2)^2 + \frac{1}{6}(x^2+y^2)^3\Big), \end{aligned} \tag{1}$$

where $\varepsilon \geq 0$ is a small parameter. Of course, when $\varepsilon = 0$ the Hamiltonian H_0 is the classical Hénon-Heiles Hamiltonian. The Hamiltonian (1) was introduced in [2].

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