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Differential equations with a given set of solutions

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A B S T R A C T

The aim of this paper is to study the following inverse problem of ordinary differential equations: For a given set of analytic functions $\omega = \{z_1(t), \ldots, z_r(t)\}$, with $z_j(t) = x_j(t) + iy_j(t)$ and $\bar{z}_j(t) = x_j(t) - iy_j(t)$ for $j = 1, \ldots, r$, defined in the open interval $I \subseteq \mathbb{R}$, we want to determine the differential equation

$$F(t, \bar{z}, z, \dot{z}, \dot{\bar{z}}, \ldots, z^{(n)}, \bar{z}^{(n)}) = 0,$$

where $z^{(j)} = \frac{d^j z}{dt^j}$ for $j = 1, \ldots, n$, in such a way that the given set of functions ω is a set of solutions of this differential equation.

1. Introduction and statement of the main results

In the theory of ordinary differential equations we can find two fundamental problems. The *direct problem* which consists in a broad sense in finding the solutions of a given ordinary differential equation, and the *inverse problem*. An inverse problem of ordinary differential equations is to find the more general differential equation satisfying a set of given properties (see [4,10,19,20]).

The inverse problem for determining the ordinary differential equations with given partial and first integrals was studied in [10]. The obtained results were applied in particular

(i) to construct Lagrangian mechanical systems with a given set of linear constraints with respect to the velocity and to obtain Hamiltonian systems with a given set of first integrals (see [11,20]),
(ii) to solve the 16th Hilbert problem for algebraic limit cycles (see [11–14,21]), and
(iii) to study the center-focus problem (see [15–17]).

In the rest of this paper we assume that all the functions are analytic in their variables, but we remark that this condition is for simplicity, although most of the results remain valid for C^r functions with a convenient $r \geq 1$.

We study the following two problems on the determination of ordinary differential equations with a given set of solutions.

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