INTEGRABILITY OF A CLASS OF N–DIMENSIONAL LOTKA–VOLTERRA AND KOLMOGOROV SYSTEMS

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Abstract. We study the integrability of an N–dimensional differential Kolmogorov systems of the form

$$\dot{x}_j = x_j \left(a_j + \sum_{k=1}^{N} a_{jk} x_k \right) + x_j \Psi(x_1, \ldots, x_N), \quad j = 1, \ldots, N,$$

where a_j, and a_{jk} are constants for $j, k = 1, \ldots, N$ and $\Psi(x_1, \ldots, x_N)$ is a homogenous polynomial of degree $n > 2$, with either one additional invariant hyperplane, or with one exponential factor. We also study the integrability of the N–dimensional classical Lotka-Volterra systems (when $\Psi(x_1, \ldots, x_N) = 0$). In particular we consider the integrability of the asymmetric May–Leonard systems.

1. Introduction and main results

For the N–dimensional nonlinear differential systems the existence of $K < N-1$ independent first integrals means that systems is partially integrable. The existence of $N-1$ independent first integrals means that the system is completely integrable, i.e. the intersection of the $N-1$ hypersurfaces obtained fixing the $N-1$ first integrals provide the trajectories of the differential system.

Polynomial differential systems of the form

$$\dot{x}_j = x_j f_j(x_1, \ldots, x_N), \quad \text{for} \quad j = 1, \ldots, N$$

is called the *N-dimensional Kolmogorov differential equations*, where $f_j = f_j(x_1, \ldots, x_N)$ is a given function for $j = 1, \ldots, N$.

We develop a method based in the study of the rank of convenient matrices in order to determine the integrability of a class of N–dimensional Kolmogorov systems of the form

$$\dot{x}_j = x_j \left(a_j + \sum_{k=1}^{N} a_{jk} x_k \right) + x_j \Psi(x_1, \ldots, x_N), \quad j = 1, \ldots, N, \tag{1}$$

where $\Psi(x_1, \ldots, x_N)$ is a homogenous polynomial of degree n, with either one additional invariant hyperplane, or with one exponential factor.

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