Gradient systems of harmonic polynomials

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Abstract

We characterize all local phase-portraits of the finite and infinite singular points of the gradient systems defined by the real harmonic polynomials in two variables.

We classify all the non-equivalent topological phase portraits of the gradient systems in the Poincaré disc defined by harmominc polynomials of degree less than five.

Keywords: harmonic polynomial, lgradient system, phase portrait.
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1. Introduction and statement of the main results

There are several papers studying the dynamics of the differential equations in the plane $\mathbb{R}^2$ which come from holomorphic functions in one complex variable, when we separate them in their real and imaginary components, see for instance the articles [1, 6, 8, 9, 10, 12, 13]. Also there are some papers studying differential equations in $\mathbb{R}^4$ coming from functions of one quaternion variable, see [2, 7, 14] and the papers quoted there.

On the other hand the gradient differential equations in the plane $\mathbb{R}^2$ defined by the gradient of a smooth function of two real variables have been studied by several authors, see for example [3, 4, 11].

The objective of this paper is to study the gradient differential equations in the plane $\mathbb{R}^2$ which come from harmonic polynomials of two real variables.

Let $\mathbb{P}_n(\mathbb{R}^2)$ be the set of *harmonic polynomials* of degree $n \geq 1$ in the real variables

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