HILBERT'S 16TH PROBLEM. WHEN DIFFERENTIAL SYSTEMS MEET VARIATIONAL PRINCIPLES.

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ABSTRACT. We provide an upper bound for the number of limit cycles that planar polynomial differential systems of a given degree may have. The bound turns out to be a polynomial of degree three in the degree of the system. The strategy brings together variational and dynamical system techniques, by transforming the task of counting limit cycles into counting critical points for a certain smooth, non-negative functional for which limit cycles are zeroes. We thus solve the second part of Hilbert's 16th problem providing a uniform upper bound for the number of limit cycles which only depends on the degree of the polynomial differential system.

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2010 *Mathematics Subject Classification.* Primary 34C07.
Key words and phrases. limit cycles, polynomial vector fields, polynomial differential systems, Hilbert's 16th problem.