QUALITATIVE STUDY OF A MODEL WITH RASTALL GRAVITY

JAUME LLIBRE[1] AND CHARA PANTAZI[2]

Abstract. We consider the Rastall theory for the flat Friedmann–Robertson–Walker universe filled with a perfect fluid that satisfies a linear equation of state. The corresponding dynamical system is a two dimensional system of polynomial differential equations depending on four parameters. We show that this differential system is always Darboux integrable. In order to study the global dynamics of this family of differential systems we classify all their non–topological equivalent phase portraits in the Poincaré disc.

1. Introduction and statement of the main results

Rastall in 1972 proposed a modified theory of gravity where the matter source is described by the energy momentum tensor as in general relativity and also by the metric of the external space. In empty space Rastall theory of gravity coincides with the classical Einstein gravity. More precisely, Rastall theory [12, 13, 14] suggests that the stress energy tensor of the source of the gravitational field should not conserved, namely it should be $T^{\mu\nu}_{;\nu} \neq 0$. Rastall proposed to take $T^{\mu\nu}_{;\nu} = \lambda R^{,\mu}$, where R is the Ricci scalar and λ is the Rastall constant parameter. Then the gravitational field equations can be written as (see also [12])

$$G_{\mu\nu} + k\lambda g_{\mu\nu} R = kT_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor and k is the gravitational constant in Rastall theory.

The case $\lambda = 0$ and $k = 8\pi$ gives rise to the classical Einsten's field equation of general relativity whenever $T^{\mu\nu}_{;\nu} = 0$, see [12].

For the Friedmann–Robertson–Walker universe and for a perfect fluid, the gravitational equations (1) can be written according to Yuan and Huang [15] like

$$\begin{aligned} 3(1-4k\lambda)H^2 - 6k\lambda\dot{H} + 3(1-2k\lambda)\frac{\kappa}{a^2} &= \ k\rho, \\ 3(1-4k\lambda)H^2 + 2(1-3k\lambda)\dot{H} + (1-6k\lambda)\frac{\kappa}{a^2} &= \ -kp, \end{aligned} \tag{2}$$

and $H = \dot{a}/a$ is the Hubble parameter. Here the dot represents time derivative. The $a(t)$ is the scale factor (dimensionless). Here $\kappa = -1, 0, +1$ for open, flat and closed spatial sections respectively. As usual ρ is the energy density and p is the pressure. Here we consider the case of flat spatial sections and so we consider that $\kappa = 0$. Hence, from the first equation of (2) and considering that $\lambda \neq 0$ and $k \neq 0$ we obtain

$$\dot{H} = -\frac{4k\lambda - 1}{2k\lambda} H^2 - \frac{1}{6\lambda}\rho. \tag{3}$$

Now the contracted Bianchi identities $G^{\mu}_{\nu;\mu} = 0$ yields to the equation of continuity (see [2])

$$(3k\lambda - 1)\dot{\rho} + 3k\lambda\dot{p} + 3(4k\lambda - 1)(\rho + p)H = 0. \tag{4}$$

A perfect fluid with linear equation of state has the following form (see the work of Babichev and collaborators [1])

$$p = \alpha(\rho - \rho_0), \tag{5}$$

where α and ρ_0 are constant parameters.