ON THE BIRTH AND DEATH OF ALGEBRAIC LIMIT CYCLES IN QUADRATIC DIFFERENTIAL SYSTEMS

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ABSTRACT. In 1958 started the study of the families of algebraic limit cycles in the class of planar quadratic polynomial differential systems. In the present we known one familiy of algebraic limit cycles of degree 2 and four families of algebraic limit cycles of degree 4, and that there are not limit cycles of degree 3. These families of degree 2 and 4 are all the families of those degrees modulo an affine change of variables and a scaling of the time. We also know that there exist two families of algebraic limit cycles of degree 5 and one family of degree 6, but we do not know if these families are all the families of degree 5 and 6. Until today it is an open problem to know if there are algebraic limit cycles of degree higher than 6 inside the class of quadratic polynomial differential systems. Here we investigate the birth and death of all the known families of algebraic limit cycles of quadratic polynomial differential systems.

1. Introduction and statement of the main results

Consider the polynomial differential system in $\mathbb{R}^2$ given by

$$\begin{aligned} \dot{x} &= P(x,y), \\ \dot{y} &= Q(x,y), \end{aligned} \tag{1}$$

where the dot denotes derivative with respect to the *time* t. The number $m = \max\{\deg P, \deg Q\}$ is the *degree* of system (1).

When $m = 2$ system (1) is a *quadratic polynomial differential system* or simply a *quadratic system*. Associated with system (1) we have the polynomial vector field

$$\mathcal{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}.$$

An isolated periodic orbit in the set of all periodic orbits of system (1) is a *limit cycle*. Although there are hundreds of papers published on quadratic systems it remains many open questions on the existence, birth and death of the limit cycles of quadratic systems. In recent years a variety of methods were used to investigate the existence of limit cycles in quadratic systems.

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