ON THE LIMIT CYCLE OF A BELOUSOV–ZABOTINSKY DIFFERENTIAL SYSTEMS

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Abstract. The authors in [7] shown numerically the existence of a limit cycle surrounding the unstable node that system (1) has in the positive quadrant for specific values of the parameters. System (1) is one of the Belousov–Zabotinsky dynamical models. The objective of this paper is to prove that system (1), when in the positive quadrant Q has an unstable node or focus, has at least one limit cycle and, when $f = 2/3$, $q = \epsilon^2/2$ and $\epsilon > 0$ sufficiently small this limit cycle is unique.

1. Introduction and statement of the main results

One of the most studied chemical oscillation systems is the Belousov-Zhabotinsky (BZ) reaction, which was elucidated by 20 chemical equations to explain the reaction mechanism and was simplified to three-variable differential equations. Many works about physical and chemical mechanism, numerical simulation and experimental research on BZ reaction appeared, see for instance [1, 3, 4, 9] . After the 1990s, the slow-fast oscillation was found in many chemical reactions the reason was that the catalyst could make the reaction process involve in different time scales with large gap. But most of the researches on the slow-fast oscillation in chemical reaction were limited to numerical simulation and experimental investigation.

One of the BZ dynamical models given as a slow-fast system is the following

$$\epsilon\dot{x} = x(1-x) + \frac{f(g-x)}{q+x}y, \qquad \dot{y} \;= x - y, \tag{1}$$

where the parameters f and q are positive and $\epsilon > 0$ is sufficiently small. As usual the dot denotes derivative with respect to the time t.

In [7] the authors shown numerically the existence of a limit cycle surrounding the unstable node that this system has in the positive quadrant $Q = \{(x, y) \in \mathbb{R}^2 : x > 0 \text{ and } y > 0\}$, for the values of the parameters $f = 2/3, \epsilon = 1/25$ and $q = \epsilon^2/3$.

The objective of this paper is to prove that system (1), when in the positive quadrant Q has an unstable node or focus, then first it has at least one limit cycle, and second, that when $f = 2/3$, $q = \epsilon^2/2$ and $\epsilon > 0$ sufficiently small this limit cycle is unique. More precisely, our main results are the following two theorems:

Theorem 1. *The BZ differential system* (1) *when for f, q and $\epsilon > 0$ has an unstable focus or node in the first quadrant Q, then it has at least one limit cycle in Q.*

Theorem 2. *The BZ differential system* (1) *for $f = 2/3$, $q = \epsilon^2/3$ and $\epsilon > 0$ sufficiently small has a unique stable limit cycle in the quadrant Q, which is the unique limit cycle of the system.*

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