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Limit cycles of piecewise polynomial perturbations of higher dimensional linear differential systems

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Abstract. The averaging theory has been extensively employed for studying periodic solutions of smooth and nonsmooth differential systems. Here, we extend the averaging theory for studying periodic solutions a class of regularly perturbed non-autonomous n-dimensional discontinuous piecewise smooth differential system. As a fundamental hypothesis, it is assumed that the unperturbed system has a manifold $\mathcal{Z} \subset \mathbb{R}^n$ of periodic solutions satisfying $\dim(\mathcal{Z}) < n$. Then, we apply this result to study limit cycles bifurcating from periodic solutions of linear differential systems, $x' = Mx$, when they are perturbed inside a class of discontinuous piecewise polynomial differential systems with two zones. More precisely, we study the periodic solutions of the following differential system:

$$x' = Mx + \varepsilon F_1^n(x) + \varepsilon^2 F_2^n(x),$$

in $\mathbb{R}^{d+2}$, where ε is a small parameter, M is a $(d+2)\times(d+2)$ matrix having one pair of pure imaginary conjugate eigenvalues, m zeros eigenvalues, and $d-m$ non-zero real eigenvalues.

1. Introduction

The analysis of discontinuous piecewise smooth differential systems has recently had a large and fast growth due to its applications in several areas of the knowledge. Such systems model many phenomena in control systems (see [1]), impact on mechanical systems (see [2]), economy (see [17]), biology (see [18]), nonlinear oscillations (see [27]), neuroscience (see [8], [13], [28]), and other fields of science.

Establishing the existence of limit cycles is one of the major problem in the theory of differential systems. The interest in detecting such objects is due to the fact that they are non-local invariant sets providing information on the qualitative behavior of the system. The first studies on this subject considered smooth

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