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Bifurcations from families of periodic solutions in piecewise differential systems

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ABSTRACT

Consider a differential system of the form

$$x' = F_0(t,x) + \sum_{i=1}^{k} \varepsilon^i F_i(t,x) + \varepsilon^{k+1} R(t,x,\varepsilon),$$

where $F_i : \mathbb{S}^1 \times D \to \mathbb{R}^m$ and $R : \mathbb{S}^1 \times D \times (-\varepsilon_0, \varepsilon_0) \to \mathbb{R}^m$ are piecewise C^{k+1} functions and T-periodic in the variable t. Assuming that the unperturbed system $x' = F_0(t,x)$ has a d-dimensional submanifold of periodic solutions with $d < m$, we use the Lyapunov–Schmidt reduction and the averaging theory to study the existence of isolated T-periodic solutions of the above differential system.

1. Introduction and statement of the main result

1.1. Introduction

The study of invariant sets, in special isolated periodic solutions, is very important for understanding the dynamics of a differential system. In the present study, we are concerned about isolated T-periodic solutions of non-autonomous differential systems written in the form

$$\begin{aligned} x' &= F(t,x;\varepsilon) \\ &= F_0(t,x) + \sum_{i=1}^{k} \varepsilon^i F_i(t,x) + \varepsilon^{k+1} R(t,x,\varepsilon), \ (t,x) \in \mathbb{R} \times D. \end{aligned} \quad (1)$$

Here, the prime denotes the derivative with respect to the independent variable t, all the functions are assumed to be T-periodic in t, D is an open subset of $\mathbb{R}^m$, and ε is a small parameter. In this regard, the averaging theory serves as an important tool to detect periodic solutions of (1). A classical introduction to the averaging theory can be found in [1,2].

There are many studies concerning the periodic solutions of system (1). As a fundamental hypothesis, it is usually assumed that the unperturbed system $x' = F_0(t,x)$ has a submanifold of initial conditions $\mathcal{Z} \subset D$ whose orbits are T-periodic. These studies differ among them depending on the regularity of system (1) and on the dimension of $\mathcal{Z}$. In what follows, we shall quote some of them.

For the case $\dim(\mathcal{Z}) = m$, the classical averaging theory [1,2] provides sufficient conditions for the existence of periodic solutions of (1) assuming $F_0 = 0$ and some smoothness and boundedness conditions. In [3], the authors extended the former results up to $k = 2$ assuming weaker conditions on the regularity of system (1). In [4], the authors dropped the condition $F_0 = 0$ and developed the averaging theory at any order ($k \geq 1$ being an arbitrary integer) assuming the analyticity of the system (1). The analyticity condition was relaxed in [5] by means of topological methods. The averaging theory was also extended to non-smooth differential systems [6–10]. The study of non-smooth differential systems is important in many fields of applied sciences since many problems of physics, engineering, economics, and biology are modeled using differential equations with discontinuous right-hand side, see for instance [11–13]. Thus, there is natural interest in studying the periodic solutions of system (1) when it is not smooth.

For the case $\dim(\mathcal{Z}) < m$, the averaging theory by itself is not enough to analyze the periodic solutions of system (1) and other techniques need to be employed with it, such as the *Lyapunov–Schmidt reduction method*. In the case that F_i's are smooth functions, we may quote the studies [14–17]. If the functions F_i are not smooth or even continuous, we have studies [8,18], where the authors analyzed some classes of these systems.

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