FIRST INTEGRALS OF A CLASS OF n-DIMENSIONAL LOTKA-VOLTERRA DIFFERENTIAL SYSTEMS

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Abstract. Lotka-Volterra model is one of the most popular in biochemistry. It is used to analyze cooperativity, autocatalysis, synchronization at large scale and especially oscillatory behavior in biomolecular interactions. These phenomena are in close relationship with the existence of first integrals in this model. In this paper we determine the independent first integrals of a family of n–dimensional Lotka-Volterra systems. We prove that when $n = 3$ and $n = 4$ the system is completely integrable. When $n \geq 6$ is even, there are three independent first integrals, while when $n \geq 5$ is odd there exist only two independent first integrals. In each of these mentioned cases we identify in the parameter space the conditions for the existence of Darboux first integrals. We also provide the explicit expressions of these first integrals.

1. Introduction and formulation of the problem

The real nonlinear ordinary differential systems are widely used to model processes or reactions in a variety of fields of science, from biology and chemistry to economy, physics and engineering. The qualitative theory of dynamical systems is employed to analyze the behavior of these dynamical systems. Within this analysis one of the important features is the existence of first integrals of the differential systems defined in $\mathbb{R}^n$. This is mainly due to the fact that the existence of a first integral allows to reduce the dimension of the system by one. So in the qualitative theory of the differential systems are important the methods allowing to detect the presence of first integrals.

In this paper we shall apply the Darboux theory of integrability to real polynomial Lotka-Volterra differential systems. This theory provides a method of constructing first integrals of polynomial differential systems, based on the number of invariant algebraic hypersurfaces that they have. Since its publication in 1878, this theory originally

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