POLYNOMIAL VECTOR FIELDS ON THE CLIFFORD TORUS

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Abstract. First we characterize all the polynomial vector fields in $\mathbb{R}^4$ which have the Clifford torus as an invariant surface. After we study the number of invariant meridians and parallels that such polynomial vector fields can have in function of the degree of these vector fields.

1. Introduction and statement of the main results

The Clifford torus

$$\mathbb{T} = \left\{ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : \ x_1^2 + x_2^2 = \frac{1}{2}, \ x_3^2 + x_4^2 = \frac{1}{2} \right\}$$

in geometric topology is the simplest and most symmetric Euclidean space embedding of the cartesian product of two circles. It lives in $\mathbb{R}^4$, as opposed to $\mathbb{R}^3$.

In MathSciNet at July 22 of 2017 it appears with the keyword "Clifford torus" 430 references. The more recent reference is [7]. In the reference [6] are studied the meridians of the surfaces of revolution and some information about the meridians of the Clifford torus can be found there. In the references [2, 12] are studied the parallels of the surfaces of revolutions and again contains some information on the parallels of the Clifford torus.

In this paper first we shall study the polynomial vector fields of arbitrary degree in $\mathbb{R}^4$ having the Clifford torus invariant by their flow, and after we shall compute the maximal number of parallels and meridians that a polynomial vector field of a given degree can exhibit on the Clifford torus.

The maximum number of invariant hyperplanes that a polynomial vector field in $\mathbb{R}^n$ can have in function of its degree was given in [8]. The

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