# PHASE PORTRAITS OF THE HIGGINS–SELKOV SYSTEM

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Abstract. We study the dynamics of the Higgins–Selkov system

$$\dot{x} = 1 - xy^{\gamma}, \quad \dot{y} = \alpha y(xy^{\gamma-1} - 1),$$

where $\alpha$ is a real parameter and $\gamma > 1$ is an integer. We classify the phase portraits of these systems for $\gamma = 3, 4, 5, 6$, in the Poincaré disc for all the values of the parameter $\alpha$. We determine in function of the parameter $\alpha$ the regions of the phase space with biological meaning.

## 1. Introduction and statement of the main results

The *glycolysis* is the first step in the transformation of glucose into energy for cellure metabolism. Higgins in [8] used a mathematical model for investigating sustained oscillations in the glycolysis process. But his model did not show any limit cycle corresponding to sustained oscillations observed in the experiments. For solving this problem Selkov in [11] introduced another mathematical model for studying the glycolysis now called the Higgins–Selkov system. This system is

$$\frac{dx}{dt} = 1 - xy^{\gamma}, \quad \frac{dx}{dt} = \alpha y(xy^{\gamma-1} - 1), \tag{1}$$

where $x$ and $y$ are state variables, $t$ is the time variable, $\alpha$ is a real parameter, and $\gamma > 1$ is an integer.

The point $(1, 1)$ is the unique singular point of system (1) for all values of parameter $\alpha \neq 0$ and $\gamma > 1$. It is shown that a supercritical Hopf bifurcation occurs at $\alpha = 1/(\gamma - 1)$. Artés et al. [2] in 2018 characterized the dynamics of system (1) with $\gamma = 2$ and $\alpha \in \mathbb{R} \setminus (1, 3)$, and proposed a conjecture about existence and uniqueness of a limit cycle when $\alpha \in (1, 3)$. Chen and Tang in [5] have proved this conjecture which completes the phase portraits of system (1) with $\gamma = 2$. Recently

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