International Journal of Bifurcation and Chaos, Vol. 30, No. 13 (2020) 2050189 (11 pages)

DOI: 10.1142/S0218127420501898

Zero–Hopf Bifurcations in Three-Dimensional Chaotic Systems with One Stable Equilibrium

Jaume Llibre
Departament de Matemàtiques,
Universitat Autònoma de Barcelona – UAB,
08193 Bellaterra, Barcelona, Catalonia, Spain
jllibre@mat.uab.cat

Marcelo Messias*
Departamento de Matemática e Computação,
Faculdade de Ciências e Tecnologia,
Universidade Estadual Paulista – UNESP,
19060-900 P. Prudente, São Paulo, Brazil
marcelo.messias1@unesp.br

Alisson de Carvalho Reinol
Departamento Acadêmico de Matemática,
Universidade Tecnológica Federal do Paraná – UTFPR,
86812-460 Apucarana, Paraná, Brazil
alissonreinol@utfpr.edu.br

Received August 17, 2019; Revised March 10, 2020

In [Molaie *et al.*, 2013] the authors provided the expressions of 23 quadratic differential systems in $\mathbb{R}^3$ with the unusual feature of having chaotic dynamics coexisting with one stable equilibrium point. In this paper, we consider 23 classes of quadratic differential systems in $\mathbb{R}^3$ depending on a real parameter a, which, for $a = 1$, coincide with the differential systems given by [Molaie *et al.*, 2013]. We study the dynamics and bifurcations of these classes of differential systems by varying the parameter value a. We prove that, for $a = 0$, all the 23 considered systems have a nonisolated zero–Hopf equilibrium point located at the origin. By using the averaging theory of first order, we prove that a zero–Hopf bifurcation takes place at this point for $a = 0$, which leads to the creation of three periodic orbits bifurcating from it for $a > 0$ small enough: an unstable one and a pair of saddle type periodic orbits, that is, periodic orbits with a stable and an unstable manifold. Furthermore, we numerically show that the hidden chaotic attractors which exist for these systems when $a = 1$ are obtained by period-doubling route to chaos.

Keywords: Zero–Hopf bifurcation; periodic orbits; hidden chaotic attractors; period-doubling route to chaos.

*Author for correspondence