



# Global Dynamics and Bifurcation of Periodic Orbits in a Modified Nosé-Hoover Oscillator

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## Abstract

We perform a global dynamical analysis of a modified Nosé-Hoover oscillator, obtained as the perturbation of an integrable differential system. Using this new approach for studying such an oscillator, in the integrable cases, we give a complete description of the solutions in the phase space, including the dynamics at infinity via the Poincaré compactification. Then using the averaging theory, we prove analytically the existence of a linearly stable periodic orbit which bifurcates from one of the infinite periodic orbits which exist in the integrable cases. Moreover, by a detailed numerical study, we show the existence of nested invariant tori around the bifurcating periodic orbit. Finally, starting with the integrable cases and increasing the parameter values, we show that chaotic dynamics may occur, due to the break of such an invariant tori, leading to the creation of chaotic seas surrounding regular regions in the phase space.

**Keywords** Nosé-Hoover oscillator · First integral · Periodic orbit · Averaging theory · Invariant tori · Chaotic dynamics

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