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ON THE LIMIT CYCLES OF A CLASS OF DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS

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Abstract. In this paper we consider discontinuous piecewise linear differential systems whose discontinuity set is a straight line L which does not pass through the origin. These systems are formed by two linear differential systems of the form $\dot{x} = Ax \pm b$. We study the limit cycles of this class of discontinuous piecewise linear differential systems. We do this study by analyzing the fixed points of the return map of the system defined on the straight line L. This kind of differential systems appear in control theory.

1. **Introduction and statement of the main results.** The theory of discontinuous piecewise differential systems is in constant development due to its applicability in different areas of the knowledge such as ecology, mechanic and electrical engineering, see for instance [6]. However even in the planar case there are important questions unsolved for this class of differential systems as to know the number of their limit cycles.

In 2010 Han and Zhang [4] conjectured that piecewise linear systems with only two regions have at most two limit cycles. In 2012 Huan and Yang [5] investigated the number of limit cycles of planar piecewise linear systems with two regions sharing the same equilibrium. Moreover they provided a numerical example to illustrate the existence of three limit cycles, thus had a negative answer to the conjecture by Han and Zhang. In 2012 Llibre and Ponce [10] provided a rigorous proof of the existence of such three limit cycles. This was the first example that a discontinuous differential piecewise linear systems with two regions can have three limit cycles.

Many others researchers have analyzed the existence of limit cycles for a piecewise linear systems with two regions separated by a straight line. In [3] it is proved that

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