Journal of Dynamics and Differential Equations
https://doi.org/10.1007/s10884-020-09825-8

The Markus–Yamabe Conjecture Does not Hold for Discontinuous Piecewise Linear Differential Systems Separated by One Straight Line

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Received: 6 August 2019 / Revised: 14 December 2019

Abstract
The Markus–Yamabe conjecture is a conjecture on global asymptotic stability. The conjecture states that if a differentiable system $\dot{x} = f(x)$ has a singularity and the Jacobian matrix $Df(x)$ has everywhere eigenvalues with negative real part, then the singularity is a global attractor. In this paper we consider discontinuous piecewise linear differential systems in $\mathbb{R}^2$ separated by one straight line Σ such that the unique singularity of the system is at Σ and the Jacobian matrix of the system has everywhere eigenvalues with negative real part. We prove that these discontinuous piecewise linear differential systems can have one crossing limit cycle and consequently these systems do not satisfy the Markus–Yamabe conjecture.

Keywords Discontinuous piecewise linear differential system · Limit cycle · Markus–Yamabe conjecture

Mathematics Subject Classification Primary 34C05 · 34C07 · 37G15

1 Introduction and Statement of the Main Results

Consider $f(x)$ a C^1 map on an n-dimensional real vector space. Let

$$\dot{x} = f(x) \tag{1}$$

be a differential system such that $f(0) = 0$. In 1960 Markus and Yamabe stated that if all eigenvalues of $Df(x)$ have negative real part, then the origin of (1) is a global attractor. In their paper [11] the conjecture has been proved under some strong additional hypotheses. This statement became known as the Markus–Yamabe conjecture.

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Published online: 28 January 2020