LIMIT CYCLES OF A PERTURBATION OF A POLYNOMIAL HAMILTONIAN SYSTEMS OF DEGREE 4 SYMMETRIC WITH RESPECT TO THE ORIGIN

JAUME LLIBRE, PAULINA MARTÍNEZ, AND CLAUDIO VIDAL

Abstract. We study the number of limit cycles bifurcating from the origin of a Hamiltonian system of degree 4. We prove using the averaging theory of order 7, that there are quartic polynomial systems close these Hamiltonian systems having 3 limit cycles.

1. Introduction and statement of the main result

One of the main open problems in the qualitative theory of planar differential systems is the determination of limit cycles. Closely related to the Hilbert's 16th problem is the study of the limit cycles from planar differential systems when we vary the parameters bifurcating from a center, or from its periodic solutions, and has been exhaustively studied in the last century. However there is no general method to solve completely this problem, the averaging theory as being largely studied in recent years in order to analyze the problem of the bifurcation of limit cycles, see for instance [4, 2, 7, 9, 8, 10, 13, 14, 17, 18, 15, 23]. For details about the averaging theory see the book of Sanders, Verhults and Murdock [21].

In this work we deal with polynomial differential systems in $\mathbb{R}^2$ of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where the dot denotes derivative with respect to an independent real variable t, usually called the *time*. Assume that the origin O is an equilibrium point of system (1). When all the orbits of system (1) in a punctured neighborhood of the equilibrium point O are periodic, we say that the origin is a *center*. The study of the centers remain open in the present days and was started by Poincaré [20] and Dulac [6].

We focus on a polynomial differential system (1) having a center at the origin of linear type, i.e. after a linear change of variables and a scaling of the time variable, it can be written in the form:

$$\dot{x} = -y + P_2(x, y), \quad \dot{y} = x + Q_2(x, y),$$

where $P_2(x, y)$ and $Q_2(x, y)$ are polynomials without constant and linear terms.

This paper is a natural continuation of the work "Linear type centers of polynomial Hamiltonian systems with nonlinearities of degree 4 symmetric with respect to the y-axis" [16] where we consider the Hamiltonian systems

$$\dot{x} = -y - x^4 - 3bx^2y^2 - 5cy^4, \quad \dot{y} = x + 4x^3y + 2bxy^3, \tag{2}$$

of degree 4 with Hamiltonian function

$$H(x, y) = \frac{1}{2}(x^2 + y^2) + x^4y + bx^2y^3 + cy^5, \tag{3}$$

and are classify all the phase portraits of these Hamiltonian systems in the Poincaré disk, see Figure 1.

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