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Research paper

Limit cycles bifurcating of Kolmogorov systems in $\mathbb{R}^2$ and in $\mathbb{R}^3$

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A B S T R A C T

In this work we consider the Kolmogorov system of degree 3 in $\mathbb{R}^2$ and $\mathbb{R}^3$ having an equilibrium point in the positive quadrant and octant, respectively. We provide sufficient conditions in order that the equilibrium point will be a Hopf point for the planar case and a zero-Hopf point for the spatial one. We study the limit cycles bifurcating from these equilibria using averaging theory of second and first order, respectively. We note that the equilibrium point is located in the quadrant or octant where the Kolmogorov systems have biological meaning.

1. Introduction and statement of the main results

The Lotka–Volterra systems, which are a class of polynomial differential systems of degree 2 in the plane, were developed independently by Alfred J. Lotka in 1925 [11] and Vito Volterra in 1926 [20], they initially proposed these models for studying the interactions between two species. Kolmogorov [8] in 1936 extended these systems to arbitrary dimension and degree, which are now called Kolmogorov systems.

The Lotka–Volterra and Kolmogorov systems have been applied to model different natural phenomena such as the time evolution of conflicting species in biology (see for more details May [15]), the evolution of competition between three species (studied by May and Leonard [14]), the evolution of electrons, ions and neutral species in plasma physics [12], chemical reactions [7], hydrodynamics [2], economics [18], etc.

We want to consider the Kolmogorov systems of degree 3 in the plane (resp. in the space) that has an equilibrium (a, b) in the plane (resp. (a, b, c) in the space) in the interior of the first quadrant (resp. octant). We note that the region of ecological interest in Lotka-Volterra and Kolmogorov systems is indeed the first quadrant (resp. octant). It is easy to see

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