

# Periodic Solutions for a Class of Non-autonomous Newton Differential Equations

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**Abstract** We provide sufficient conditions for the existence of periodic solutions of the second-order non-autonomous differential equation

$$\ddot{x} = -\nabla_x V(t, x),$$

in  $\mathbb{R}^n$ , where  $V(t, x) = \frac{\|x\|^2}{2} + \varepsilon W(t, x)$  with  $W(t, x)$  a  $2\pi$ -periodic function in the variable  $t$ ,  $\varepsilon$  is a small parameter,  $x \in \mathbb{R}^n$  and

$$\nabla_x V(t, x) = \left( \frac{\partial V}{\partial x_1}, \dots, \frac{\partial V}{\partial x_n} \right).$$

Note that this is a particular class of non-autonomous Newton differential equations. Moreover we provide some applications.

**Keywords** Periodic solution · Newton differential equation · Averaging theory

**Mathematics Subject Classification** 37G15 · 37C80 · 37C30

## Introduction and Statement of the Main Results

In this paper we shall study the existence of periodic solutions of the second-order non-autonomous differential equation in  $\mathbb{R}^n$  of the form

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