ZERO–HOPF PERIODIC ORBITS FOR A RÖSSLER DIFFERENTIAL SYSTEM

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ABSTRACT. We study the zero-Hopf bifurcation of the Rössler differential system

$$\dot{x} = x - xy - z, \quad \dot{y} = x^2 - ay, \quad \dot{z} = b(cx - z).$$

where the dot denotes derivative with respect to the independent variable t and a, b, c are real parameters.

1. Introduction and statement of the main result

Rössler, using the geometry of the 3-dimensional flows, introduced several differential systems as prototypes of the simplest autonomous differential equations exhibiting chaos. The simplicity of his systems is in the sense of minimal dimension, minimal number of parameters and minimal nonlinearities. Nowadays in MathSciNet appear more than 114 articles with the words Rössler system in the title. In 2006 Letellier, Roulin and Rössler [2] did a classification of chaotic attractors in $\mathbb{R}^3$ in their classification they go back to the differential system

$$\begin{aligned} \dot{x} &= x - xy - z, \\ \dot{y} &= x^2 - ay, \\ \dot{z} &= b(cx - z), \end{aligned} \tag{1}$$

already considered by Rössler [2, 5, 6] in 1977, showing that this system exhibits a pure cut chaos in their terminology. As usual the dot denotes derivative with the time t. This differential system has three families of zero-Hopf equilibria. Our objective is to study if from these equilibria it bifurcate some periodic orbits.

Our interest in system (1) is motivated due to the fact that frequently the complex dynamics of some chaotic nonlinear systems started in their equilibria. More precisely, Cândido and Llibre in [1] studied the existence of zero-Hopf bifurcations in 3-dimensional systems, and numerically they show that such bifurcations sometimes are the starting bifurcation of a route to the chaotic motion.

Note that system (1) is invariant under the symmetry $(x, y, z) \to (-x, y, -z)$. A *zero-Hopf equilibrium* is an equilibrium point of a 3-dimensional autonomous differential system, which has a zero eigenvalue and a pair of purely imaginary eigenvalue.

Proposition 1. *There are two one-parameter families of systems (1) for which this system has a zero-Hopf equilibrium point namely the origin when* $a = 0, b = 1, c =$

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