

# LIMIT CYCLES OF NONLINEAR TORSIONAL VIBRATION SYSTEM VIA AVERAGING THEORY

JAUME LLIBRE<sup>1</sup> AND AMAR MAKHLOUF<sup>2</sup>

ABSTRACT. We study the existence of limit cycles of the nonlinear torsional vibration equation of the form

$$\ddot{x} + \omega^2 x + \alpha x^2 + \beta x^3 + \gamma \dot{x} = G(t),$$

where  $G(t)$  is a  $\frac{2\pi}{\omega}$ -periodic function in the variable  $t$ , and  $\alpha, \beta, \gamma, \omega$  are arbitrary parameters. Moreover we provide an application.

## 1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

In rotating machinery equipments appear widely torsional vibration systems, for instance in rolling mill, turbine generator and steam turbine. Torsional vibration may be due to unbalanced rotating parts, or due to torque fluctuations, or due to other mechanical reasons. If these vibrations are not controlled may cause damage or destruction to the rotating axes or their complements. These torsional vibrations are important on the performance and the reliability of a mechanical drive system. Consequently to study the stability or instability of torsional vibration mechanisms and their dynamics behaviors are important for the optimal design and vibration monitoring of a system.

Nonlinear torsional vibration systems have been studied intensively in these last years see for instance [2, 3, 4, 8]. But the strongly nonlinear torsional vibration systems became each time more important in engineering, and its dynamics behaviors have received less attention. In [10] using Lagrange equations of motion the authors obtained the dynamics equation of strongly nonlinear torsional vibration systems with external excitation and a quadratic and cubic nonlinear rigidity. They studied bifurcations and chaotic motions on these systems.

Our aim is to study the periodic solutions of the nonlinear torsional vibration systems considered in [10]. More precisely, we want to study

---

2010 *Mathematics Subject Classification.* 37G15, 37C80, 37C30.

*Key words and phrases.* Limit cycle, nonlinear torsional vibration system, averaging theory.